

# Conceptual Problems of the Standard Cosmological Model

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## Abstract

The physics of the expansion of the universe is still a poorly studied subject of the standard cosmological model. This because the concept of expanding space can not be tested in the laboratory and because “expansion” means continuous creation of space, something that leads to several paradoxes. We re-consider and expand here the discussion of conceptual problems, already noted in the literature, linked to the expansion of space. In particular we discuss the problem of the violation of energy conservation for local comoving volumes, the exact Newtonian form of the Friedmann equations, the receding velocity of galaxies being greater than the speed of light, and the Hubble law inside inhomogeneous galaxy distribution. Recent discussion by Kiang, Davis & Lineweaver, and Whiting of the non-Doppler nature of the Lemaitre cosmological redshift in the standard model is just a particular consequence of the paradoxes mentioned above. The common cause of these paradoxes is the geometrical description of gravity (general relativity), where there is not a well defined concept of the energy-momentum tensor for the gravitational field and hence no energy-momentum conservation for matter plus gravity.

## 1 The “Absurd Universe” of Modern Cosmology

In a number of recent papers [28,29,30], one of which entitled “Absurd universe”, Michael Turner emphasized the unpleasant status of the most widely accepted cosmological model, where about 95 % of the total matter density of the universe have unknown physics. This so called “consensus universe” being dominated by cold dark matter and dark energy, two hypothetical entities not at all tested in the laboratory.

Beside this problem with the “dark sector” of the universe, the situation is made even worse when one considers the severe conceptual problems of the standard cosmological model arising even within the boundaries of the known physics. These problems were first discovered and analyzed by Edward Harrison (1981, 1993, 1995)[13, 14, 15] but are poorly known and very rarely discussed in the literature. Here we consider a number of puzzling properties of Friedmann expanding models: the violation of energy-momentum conservation for any local comoving ball with non-zero pressure, the exactly Newtonian form of the relativistic Friedmann equations, the unlimited receding velocities

of galaxies, and the linear Hubble law inside strongly inhomogeneous galaxy distribution. Existence of these paradoxes means that the standard model of the universe is much more "absurd" than one usually thinks.

In this report an account of the origin of these conceptual problems is given and a common cause for their occurrence is proposed.

## 2 Two-fluid matter-dark energy FLRW model

The two basic ingredients of modern cosmological models are:

- *relativistic theories of gravity, and*
- *the cosmological principle.*

For the Friedmann-Lemaitre-Robertson-Walker (FLRW) model, which is the currently accepted basis for the interpretations of all astrophysical observations and the basis of the Standard Cosmological Model (SCM), these ingredients are the general relativity and Einstein's cosmological principle. Modern versions of SCM make important distinction between usual matter (with positive pressure) and dark energy (with negative pressure). A general classification and the main properties of the two-fluid FLRW models were recently discussed by Gromov et al.(2004)[11].

### 2.1 General Relativity

The first fundamental element of the SCM is the General Relativity (GR), which is a geometrical gravity theory. GR was successfully tested in the weak gravity condition of the Solar System and binary neutron stars. It is assumed that GR can be applied to the Universe as a whole.

According to GR gravity is described by a metric tensor  $g^{ik}$  of a Riemannian space. The "field" equations in GR (Einstein-Hilbert equations) have the form (we use Landau & Lifshitz 1971 [19] notations):

$$\mathfrak{R}^{ik} - \frac{1}{2} g^{ik} \mathfrak{R} = \frac{8\pi G}{c^4} (T_{(m)}^{ik} + T_{(de)}^{ik}) \quad (1)$$

where  $\mathfrak{R}^{ik}$  is the Ricci tensor,  $T_{(m)}^{ik}$  is the energy-momentum tensor (hereafter EMT) for usual matter, and  $T_{(de)}^{ik}$  is the dark energy component, which includes the famous cosmological constant.

From the Bianchi identity one gets the continuity equation in the form:

$$T_{k ; i}^i = (T_{(m)}^i{}_k + T_{(de)}^i{}_k) ; i = 0 \quad (2)$$

where  $T_k^i$  is the total EMT of the matter and dark energy. In the case of non-interacting matter and dark energy the divergence of each EMT equals zero separately. The general case of energy transfer between matter and dark energy was considered by Gromov et al. 2004 [11].

Note that gravity in GR is not at all equivalent to matter, so the total EMT  $T^{ik}$  does not contain the EMT of gravity field. This is why Eq.(2) is not a conservation law for gravity plus total matter (Landau & Lifshitz 1971 [19], sec.101, p.304).

## 2.2 Einstein's Cosmological Principle

The second element of the SCM is the Einstein's Cosmological Principle. This states that the universe is spatially homogeneous and isotropic on "large scales" (see e.g. Weinberg 1972 [31]; Peebles 1993 [23]; Peacock 1999 [22]). Here the term "large scales" relates to the fact that the universe is certainly inhomogeneous at scales of galaxies and clusters of galaxies. Therefore, the hypothesis of homogeneity and isotropy of the matter distribution in space means that starting from  $r_{hom}$ , at all scales  $r > r_{hom}$  we can write the total energy density  $\varepsilon = \rho c^2$  and the total pressure  $p$  as a function of time only:

$$\varepsilon(\vec{r}, t) = \varepsilon(t) \quad (3)$$

$$p(\vec{r}, t) = p(t) \quad (4)$$

where the total energy density is the sum of the energy density of ordinary matter ( $\varepsilon_m$ ) and dark energy ( $\varepsilon_{de}$ ), and the total pressure is the sum of corresponding components:

$$\varepsilon = \varepsilon_m + \varepsilon_{de}, \quad p = p_m + p_{de} \quad (5)$$

Here usual matter has equation of state

$$p_m = \beta \varepsilon_m, \quad 0 \leq \beta \leq 1, \quad (6)$$

and dark energy has equation of state

$$p_{de} = w \varepsilon_{de}, \quad -1 \leq w < 0. \quad (7)$$

Recently values  $w < -1$  also were considered.

With these equations of general relativity and expressions for Cosmological Principle we are ready now to investigate the properties of the standard cosmological model.

## 2.3 Space expansion paradigm

An important consequence of homogeneity and isotropy is that the line element may be presented in the Robertson-Walker form:

$$ds^2 = c^2 dt^2 - S(t)^2 d\chi^2 - S(t)^2 I_k(\chi)^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (8)$$

where  $\chi, \theta, \phi$  are the "spherical" comoving space coordinates,  $t$  is synchronous time coordinate,  $I_k(\chi) = \sin(\chi), \chi, \sinh(\chi)$  corresponding to curvature constant values  $k = +1, 0, -1$  respectively and  $S(t)$  is the scale factor.

In the *expanding space paradigm* the proper metric distance  $r$  of a body with fixed comoving coordinate  $\chi$  from the observer is given by

$$r = S(t) \cdot \chi \quad (9)$$

and increases with time  $t$  as the scale factor  $S(t)$ . Note that physical dimension of metric distance  $[r] = cm$ , hence if  $[S] = cm$  then  $\chi$  is the dimensionless comoving coordinate distance. In fact  $\chi$  is the spherical angle and  $S(t)$  is the radius of the sphere (or pseudosphere) embedded in the 4-dimensional Euclidean space. It means that the "cm" (the measuring rod) itself is defined as unchangeable unit

of length in the embedding Euclidean space. Hence the distance  $r$  measured in  $cm$  is the "internal" proper distance on the 3-dimensional hypersurface of the embedding space. In other words  $r$  and  $\chi$  give the Eulerian and Lagrangian representation of the comoving distance.

Often, "cylindrical" comoving space coordinates  $\mu, \theta, \phi$  are used in the literature. In this case the line element is

$$ds^2 = c^2 dt^2 - S(t)^2 \frac{d\mu^2}{1 - k\mu^2} - S(t)^2 \mu^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (10)$$

The metric distance  $l$  is

$$l = S(t) \cdot \mu, \quad (11)$$

which can be interpreted as the "external" distance from z-axis in an embedding Euclidean 4-dimensional space. So it is important to use different designations for different distances defined by intervals in Eq.8 and Eq.10 (not as in Peacock 1999 [22], p.70).

The relation between these two metric distances is

$$r = S(t) I_k^{-1}(l/S) \quad (12)$$

where  $I_k^{-1}$  is the inverse function for  $I_k$ .

It is important to point out that the hypothesis of homogeneity of space implies that for a given galaxy the recession velocity is proportional to distance. The *exact relativistic* expression for the recession velocity  $V_{exp}$  of a body with fixed  $\chi$ , which due to the "space expansion" is the rate of increasing of the metric distance  $r$  as a function of time, immediately follows from Eq.9 :

$$V_{exp} = \frac{dr}{dt} = \frac{dS}{dt} \chi = \frac{dS}{dt} \frac{r}{S} = H(t)r = c \frac{r}{r_H} \quad (13)$$

where  $H(t) = \dot{S}/S$  is the Hubble constant (also is a function of time) and  $r_H = c/H(t)$  is the Hubble distance at the time  $t$ . (Here and in the following the dot indicates derivative with respect to the time  $d/dt$ .) This means that the linear velocity-distance relation  $V = Hr$ , identified with the observed Hubble law, can exist only if the matter distribution is uniform. However, according to modern data on galaxy distribution, this seems not to be the case at least for luminous matter.

## 2.4 Friedmann's equations

In comoving coordinates the total EMT has the form:

$$T_i^k = diag(\varepsilon, -p, -p, -p) \quad (14)$$

In the case of unbounded homogeneous matter distribution (Eqs.3,4) the Einstein's equations (Eq.1) are directly reduced to the Friedmann's equations. From the initial set of 16 equations we have only two independent equations for the (0,0) and (1,1) components, which may be written in the following form:

$$\frac{\dot{S}^2}{S^2} + \frac{kc^2}{S^2} = \frac{8\pi G}{3c^2} \varepsilon, \quad (15)$$

$$2 \frac{\ddot{S}}{S} + \frac{\dot{S}^2}{S^2} + \frac{kc^2}{S^2} = -\frac{8\pi G}{c^2} p. \quad (16)$$

From the Bianchi identity (Eq.2) it follows the continuity equation

$$3\frac{\dot{S}}{S} = -\frac{\dot{\varepsilon}}{\varepsilon + p}, \quad (17)$$

which must be added to the Eqs.(15), (16) as the consistency condition.

Using the definition of the Hubble constant  $H = \dot{S}/S$ , we rewrite Eq.(15) as

$$H^2 - \frac{8\pi G}{3}\varrho = -\frac{kc^2}{S^2}, \quad (18)$$

and equation (16) as

$$\ddot{S} = -\frac{4\pi G}{3}\left(\varrho + \frac{3p}{c^2}\right)S. \quad (19)$$

In terms of the critical density  $\varrho_{crit} = 3H^2/8\pi G$ , the total density parameter  $\Omega = \varrho/\varrho_{crit}$ , the curvature density parameter  $\Omega_k = kc^2/S^2H^2$ , and the deceleration parameter  $q = -\ddot{S}S/\dot{S}^2$ , these equations also may be presented in the form:

$$1 - \Omega = -\Omega_k, \quad (20)$$

$$q = \frac{1}{2}\Omega\left(1 + \frac{3p}{\varrho c^2}\right), \quad (21)$$

where  $\Omega, p, \varrho$  are the total quantities, i.e. the sum of corresponding components for matter and dark energy.

Solving the Friedmann's equation (Eq.21) one finds the dependence on time the scale factor  $S(t)$  or the metric distance  $r(t)$ .

## 3 Physics of space expansion

### 3.1 What does space expansion mean physically?

The FLRW model gives an exact mathematical description of the expanding space in the case of a geometrical theory of gravity (GR). Increasing the scale factor  $S(t)$  in FLRW metrics physically corresponds to expanding space, that is adding vacuum, and homogeneous matter. Each comoving finite box in expanding universe continuously increases its volume, so gets more and more cubic centimeters. Physically expansion of the universe means the creation of space together with physical vacuum. Creation of space may be visualized by 2-d analogy with expanding sphere in 3-d space, where the surface of the sphere increases with time and for 2-d beings their universe grows with time (gets more square centimeters) .

Real Universe is not homogeneous, it contains atoms, planets, stars, galaxies. Bondi (1947) [5] considered spherical inhomogeneities in the framework of GR and showed that inside them the space expands slowly. In fact bounded physical objects like particles, atoms, stars and galaxies do not expand. So inside these objects there is no space creation. This is why the creation of space is a new cosmological phenomenon, which is not and cannot be tested in laboratory because the Earth, the Solar System and the Galaxy do not expand.

There are several puzzling properties of FLRW models which are a direct consequence of the above derived exact equations.

### 3.2 Violation of conservation laws in expanding space

Landau & Lifshitz 1971 [19] in sec.101 "The energy-momentum pseudotensor" emphasized that equation  $T_k^i;_i = 0$  "does not generally express a law of conservation", because of the mathematical structure of the covariant divergence in Riemannian space. To get the total (all kinds of matter plus gravity) energy-momentum conserved, they suggest to consider energy-momentum pseudotensor, which could describe gravity itself. However this violates the tensor character of the laws of conservation and does not solve the problem of the energy density of the gravitational field in a geometrical description of gravity. The root of the problem lies in the equivalence principle and in the absence of a true gravity force in GR, while all other fundamental fields have true forces, true EMTs and operate in Minkowski space. It is important that Noether theorem relates conserved EMT of material fields to maximal symmetry of the Minkowski space and this is why in curved Riemannian space the EMT of gravity field can not be properly defined.

The problem of the absence of true EMT for gravity field in cosmology appears as the violation of energy conservation during the space expansion. Indeed, let us consider the energy content of a comoving ball with radius  $r(t) = S(t)\chi$ . The volume element in metric Eq.(8) is

$$dV = S^3 I_k^2(\chi) \sin(\theta) d\chi d\theta d\phi, \quad (22)$$

and energy in the comoving sphere is

$$e(r) = \int_0^r T_0^0 dV = \frac{4\pi}{3} \varepsilon(t) S^3(t) \chi^3 \sigma_k(\chi), \quad (23)$$

where  $\sigma_k(\chi) = \int_0^\chi I_k^2(y) dy$ , so that it is equal to 1 for  $k = 0$ , to  $\frac{3}{\chi^3} \left( \frac{\chi^2}{2} - \frac{\sin 2\chi}{4} \right)$  for  $k = 1$ , and to  $\frac{3}{\chi^3} \left( \frac{\sinh 2\chi}{4} - \frac{\chi^2}{2} \right)$  for  $k = -1$ .

To calculate the time dependence of the energy density we use the continuity equation (Eq.17) in the form

$$\dot{\varepsilon} = -3(\varepsilon + p) \frac{\dot{S}}{S}. \quad (24)$$

For an ideal equation of state  $p = \gamma \varrho c^2$  this equation has the simple solution

$$\varrho \propto S^{-3(1+\gamma)}, \quad (25)$$

where  $S(t)$  is the scale factor. So in particular we have for dust, radiation and vacuum

$$\varrho_{dust} \propto S^{-3}, \quad \varrho_{rad} \propto S^{-4}, \quad \varrho_{vac} \propto const. \quad (26)$$

Hence the energy inside a comoving ball will change with time as

$$e(r) = \frac{4\pi}{3} \varrho c^2 r^3 \sigma_k(\chi) \propto S^{-3\gamma}(t), \quad (27)$$

so that for dust, radiation and vacuum we get

$$e_{dust}(r) \propto const, \quad e_{rad}(r) \propto S^{-1}, \quad e_{vac}(r) \propto S^{+3}. \quad (28)$$

Intriguingly the continuity equation (Eq.24) can be written also in the form

$$dE + p dV = 0, \quad (29)$$

where  $dE = d(\varepsilon V) = d(\varrho c^2 V)$  is the change of energy within the comoving volume  $V = \text{const} \cdot S^3$ . Interestingly, Eq.(29) looks like the law of conservation of energy in thermodynamics. There is, however, an essential difference with the cosmological case.

Eq.(29) in the laboratory means that if inside a finite box the energy decreases, it reappears outside the box as the work produced by the pressure acting on a piston of a machine, increasing the volume of the box. The work performed by the pressure inside the box is the cause of the energy decrease in the box.

In cosmology Eq.(29) gives us the possibility to calculate of how much the energy increases or decreases inside a finite comoving volume but it does not tell us where the energy comes from or where it goes. This is because the cosmological pressure does not produce work. It was noted by Harrison(1981; 1995) [13, 15] that in a homogeneous unbounded expanding FLRW model one may imagine the whole universe partitioned into macroscopic cells, each of comoving volume  $V$ , and all having contents in identical states. The  $-p dV$  energy lost from any one cell cannot reappear in neighboring cells because all cells experience identical losses. So the usual idea of an expanding cell performing work on its surroundings cannot apply in this case. As Edward Harrison emphasized: "The conclusion, whether we like it or not, is obvious: energy in the universe is not conserved" (Harrison, 1981 [13], p.276).

The same conclusion was reached by Peebles (1993) [23] when he considered the energy loss inside a comoving ball of the photon gas (see our Eq.28). On page 139 he wrote "The resolution of this apparent paradox is that while energy conservation is a good local concept, ... there is not a general global energy conservation in general relativity."

In fact, only for dust ( $p = 0$ ) one may speak about energy conservation in expanding universe. But for any matter with  $p \neq 0$  within any local comoving volume energy is not conserved. This is because in GR there is no EMT of gravity field and there is no gravity force in usual physical sense.

### 3.3 Newtonian form of the relativistic Friedmann equation

Let us write Friedmann's Eq.(16) in the form

$$\frac{d^2 S}{dt^2} = -\frac{4\pi G}{3} S \left( \varrho + \frac{3p}{c^2} \right) \quad (30)$$

Because of Lagrangian comoving coordinates do not depend on time, one may rewrite Eq.30 using Eq.9 as

$$\frac{d^2 r}{dt^2} = -\frac{GM_g(r)}{r^2} \quad (31)$$

where the gravitating mass  $M_g(r)$  of a comoving ball with radius  $r$  is given by

$$M_g(r) = \frac{4\pi}{3} \left( \varrho + \frac{3p}{c^2} \right) r^3 \quad (32)$$

Friedmann's equation (Eq.31) in fact presents the cosmological Friedmann force acting on a test galaxy with mass  $m$  placed at distance  $r$  from a fixed point at the center of coordinate system:

$$F_{Fr}(r) = m \frac{d^2 r}{dt^2} = -\frac{GmM_g(r)}{r^2} \quad (33)$$

Therefore the exact relativistic equation describing the dynamical evolution of the universe is exactly equivalent to the non-relativistic Newtonian equation of motion of a test particle in the gravity field of a finite sphere containing a mass  $M_g$  within the radius  $r$ . The second term in Eq.32 does not change the Newtonian character of the solutions.

Such a similarity was first mentioned by Milne(1934) [21] and McCrea & Milne(1934) [20], though they consider the Newtonian model an approximation to Friedmann model. Later many authors claimed that the Newtonian model can be used only for small radius compared to the horizon distance. Here, however, we see that the Newtonian form of the Friedmann equation is exact and true for all radius. This creates a problem in cosmology because Eq.33 places neither such relativistic restrictions as motion velocity less than velocity of light, nor retardation response effect.

The root of the puzzle lies in the geometrical description of gravity in GR and in the derivation of Friedmann's equation from Einstein's gravity equations, using the comoving synchronous coordinates with universal cosmic time  $t$  and homogeneous unbounded matter distribution.

The Newtonian form of the Friedmann equation also creates the so called *Friedmann-Holtsmark paradox*. According to the Friedmann equation there is the cosmological force Eq.(33) acting on a galaxy situated at the distance  $r$  from another fixed galaxy. This is in apparent contradiction with well known Holtsmark result for the probability density of the force acting between particles in infinite Euclidean space in the case of  $1/r^2$  behavior of the elementary force (Holtsmark,1919 [16]; Chandrasekhar,1941 [6]). For symmetry reasons, due to the isotropy of the distribution of particles the average force in any given location is equal to zero and one is left with the finite value of fluctuating force, which is determined by the nearest neighbor particles. Hence in infinite Euclidean space with homogeneous Poisson distribution and Newtonian gravity force there is no global expansion or contraction, but there is the density and velocity fluctuations caused by local gravity force fluctuations.

Finally, the Newtonian form of the Friedmann equation explains why recession velocities of distant galaxies can be larger than the speed of light – in Newtonian theory there is no limiting velocity. The exact relativistic velocity - distance relation is the Eq.13 and it is linear for all distances  $r$ . It means that for  $r > r_H$  we get  $V_{exp} > c$  and the question arises why general relativity violates special relativity. The usual answer is that the space expansion velocity is not ordinary velocity of a body in space, hence it has no ordinary limit by the velocity of light. Again it demonstrates the unusual physics of the expanding space.

### 3.4 Continuous creation of gravitating mass

Puzzling properties of the FLRW model also come from consideration of the active gravitating mass of the cosmological fluid, which may be either positive or negative and changes sign with the cosmic time  $t$ . In the case of one fluid with equation of state  $p = \gamma \rho c^2$  the active gravitating mass (Eq.32) will be

$$M_g(r) = +\frac{4\pi}{3}(1 + 3\gamma)\rho r^3 \propto S^{-3\gamma}(t). \quad (34)$$

So for dust, radiation and vacuum we get

$$M_{dust}(r) = +\frac{4\pi}{3}\rho_{dust}r^3 \propto const(t), \quad (35)$$

$$M_{rad}(r) = +\frac{4\pi}{3}2\rho_{rad}r^3 \propto S^{-1}(t), \quad (36)$$

$$M_{vac}(r) = -\frac{4\pi}{3}2\rho_{vac}r^3 \propto -S^{+3}(t). \quad (37)$$

Hence for the case of dust the gravitating mass does not depend on time, but in the case of radiation the gravitating mass continuously disappear in the expanding universe. The most strange example is the vacuum, where the gravitating mass is negative (no such examples in lab physics). This means that vacuum antigravity continuously increases in time due to the continuous creation of gravitating (actually "antigravitating") vacuum mass. In this sense the continuous creation of matter in the Steady State cosmological model is just a particular case of the new physics of the expanding space.

## 4 Cosmological redshift in expanding space

Harrison (1981; 1993) [13, 14] clearly demonstrated that the cosmological redshift due to the expansion of the universe is a new physical phenomenon and is not the well known Doppler effect. Recently this subject was intensively discussed by Kiang (2003) [18], Davis & Lineweaver (2003) [7] and Whiting (2004) [32] in an attempt to clarify some "common big bang misconceptions" and the "expanding confusions" widely spreaded in the literature.

### 4.1 Lemaitre effect

In the SCM the cosmological redshift is a new physical phenomenon due to the expansion of space, which induce the wave stretching of the traveling photons via the Lemaitre's equation:

$$(1+z) = \frac{\lambda_0}{\lambda_1} = \frac{S_0}{S_1} \quad (38)$$

where  $z$  is cosmological redshift,  $\lambda_1$  and  $\lambda_0$  are the wavelengths at the emission and reception, respectively, and  $S_1$  and  $S_0$  the corresponding values of the scale factor. Equation (38) may be obtained from the radial null-geodesics ( $ds = 0$ ,  $d\theta = 0$ ,  $d\phi = 0$ ) of the FLRW line element.

The cosmological redshift is caused by the Lemaitre effect, which is different from the familiar Doppler effect. It is clear from comparison between relativistic Doppler and cosmological FLRW velocity-redshift  $V(z)$  relations. To get  $V(z)$  in SCM one should consider first  $V(r)$  and  $r(z)$  relations. Exact velocity-distance relation in FLRW model is Eq.13:

$$V_{exp} = H(t)r(z). \quad (39)$$

where  $H(t) = \dot{S}/S$  is the Hubble constant at the time  $t$ . The exact distance-redshift  $r(z)$  relation in FLRW model is:

$$r(t_0, z) = r(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{h(z')}, \quad (40)$$

where  $h(z)$  is taken from Friedmann equation (Eq.18)

$$h(z) = \sqrt{\tilde{\rho}(z)\Omega^0 + (1 - \Omega^0)(1+z)^2}, \quad (41)$$

where  $\Omega^0 = \rho_{tot}^0/\rho_{crit}^0$  is the density parameter in present epoch,  $\tilde{\rho}(z) = \rho_{tot}/\rho_{tot}^0$  is the normalized density of the total substances.

Analytical expressions for  $r(z)$  may be obtained only in some simple cases. For the dust universe this relation was firstly derived by Mattig in 1958 and in terms of the internal metric distance (Eqs.9, 12) it has the form:

$$r(z) = S_0 I_k^{-1} \left[ \sqrt{\frac{2q_0 - 1}{k}} \frac{zq_0 + (q_0 - 1)((2q_0z + 1)^{1/2} - 1)}{q_0^2(1 + z)} \right], \quad (42)$$

where scale factor  $S(t = t_0) = S_0$  is

$$S_0 = \frac{c}{H_0} \sqrt{\frac{k}{\Omega^0 - 1}}. \quad (43)$$

Now we can compare the exact FLRW  $V_{exp}(z)$  relation with exact relativistic Doppler  $V_{Dop}(z)$  relation:

$$V_{exp}(z) = c \frac{r(z)}{r_{Ho}}, \quad (44)$$

$$V_{Dop}(z) = c \frac{2z + z^2}{2 + 2z + z^2}. \quad (45)$$

Clearly these are two different mathematical formulae which corresponds to two different physical phenomena – Lemaitre and Doppler effects. Eqs.(44, 45) give the same results only in the first order of  $V/c$ , however the physics of space expansion is different from motion in static space.

## 4.2 Cosmological gravitational frequency shift

In 1947 in the classic paper "Spherically symmetrical models in general relativity" by Sir Hermann Bondi it was shown that, at least for small redshifts, the total cosmological redshift of a distant body may be expressed as the sum of two effects: the velocity shift (Doppler effect) due to the relative motion of source and observer, and the global gravitational shift (Einstein effect) due to the difference between the potential energy per unit mass at the source and at the observer. It means that the spectral shift depend on the distribution of matter in the space around the source. In the case of small distances Bondi derived a simple formula for redshift which is simply the sum of Doppler and gravitation effects, and which explicitly showed that "the sign of the velocity shift depends on the sign of  $v$ , but the Einstein shift is easily seen to be towards the red" (Bondi, 1947 [5], p.421). Hence according to Bondi the cosmological gravitational frequency shift is redshift (contrary to Peacock 1999 [22], p.619 and Zeldovich & Novikov 1984 [33] p.97 considerations).

It was shown by Baryshev et al.(1994) [1] that from Mattig's relation (Eq.42) it follows directly for the case of  $z \ll 1$ ,  $v/c \approx x = r/r_H$  that

$$z_{cos} \approx x + \frac{1 + q_0}{2} x^2 = \left( \frac{v}{c} + \frac{1}{2} \frac{v^2}{c^2} \right) + \frac{q_0}{2} x^2 \quad (46)$$

is the sum of Doppler and gravitational redshifts:

$$z_{cos} \approx z_{Dop} + z_{grav} \quad (47)$$

where the cosmological gravitational redshift is

$$z_{grav} = \frac{\Delta\varphi(r)}{c^2} = \frac{1}{2} \frac{GM(r)}{c^2 r} = \frac{1}{4} \Omega_0 x^2. \quad (48)$$

Note that the Eq.(48) describes the global gravitational shift due to the whole mass within the ball having the center at the source and the radius equal to the distance between the source and the observer. Hence cosmological gravitational shift depends on the whole matter distribution between the source and the observer (and should not be confused with the local gravitational shift at the source).

It is important that the center of the ball is placed at the source. Then the cosmological gravitational redshift is consistent with the causality principle according to which the event of emission of a photon by the source (which marks the centre of the ball) must precede the event of detection of the photon by an observer on the surface of the ball. The detection event marks the spherical edge of the ball, where all potential observers are situated.

In the literature there are a few discussions of the cosmological gravitational shift but they contain mistaken claims. For instance, if one consider the observer at the center of the cosmological ball and a galaxy at the edge of the sphere, then one may conclude that cosmological gravitational shift is blueshift (see Zeldovich & Novikov, 1984 [33], p.97). Also one should use proper metric distance for calculation the mass within a ball, instead of angular distance used in Peacock, 1999 [22], problem 3.4.

Note that in the case of the fractal matter distribution with fractal dimension  $D = 2$  the cosmological gravitational redshift gives the linear distance-redshift relation and becomes an observable cosmological phenomenon (see e.g. Baryshev et al. 1994 [1]).

### 4.3 Hubble-deVaucouleurs paradox

According to SCM the linear Hubble law is a consequence of the homogeneity of the matter distribution. However studies of the 3-dimensional local galaxy Universe have shown that at least in the range of scales  $\sim 1 \div 100 \text{ Mpc}$  galaxy distribution is strongly inhomogeneous and has fractal properties (Sylos Labini et al., 1998 [25]; Baryshev & Teerikorpi 2005 [4]). This confirms de Vaucouleurs' prescient view on the matter distribution so we call it de Vaucouleurs law of large scale galaxy distribution (Baryshev et al. 1998 [2]; Baryshev & Teerikorpi 2002 [3]).

At the same time modern observations of the local Hubble flow based on Cepheid distances to local galaxies, Tully-Fisher distances from the KLUN program, and other distance indicators, demonstrate that the linear Hubble law is well established within the Local Volume ( $r < 10 \text{ Mpc}$ ), starting from distances as small as 1 Mpc (see Teerikorpi, 1997 [26]; Ekholm et al., 2001 [8]; Karachentsev et al. 2003 [17]; Teerikorpi et al. 2005 [27]).

A puzzling conclusion is that the strictly linear redshift-distance relation is observed just inside inhomogeneous galaxy distribution, i.e. deep inside the fractal structure for distances less than homogeneity scale (it is known that  $r_{hom} > 30 \text{ Mpc}$ ):

$$(r < r_{hom}) \ \& \ (cz = H_0 r) \quad (49)$$

This empirical fact presents a profound challenge to the standard model where the homogeneity is the basic explanation of the Hubble law, and "the connection between homogeneity and Hubble's law was the first success of the expanding world model" (Peebles et al., 1991 [24]). In fact, within the SCM one would not expect any neat relation of proportionality between velocity and distance for close galaxies, which are members of large scale structures. However, contrary to the expectation, modern data show a good linear Hubble law even for nearby galaxies. It leads to a new observationally established puzzling fact that the linear Hubble law is not a consequence of the homogeneity of visible matter, just because the visible matter is distributed inhomogeneously.

The Hubble and de Vaucouleurs laws describe very different aspects of the Universe, but both have in common universality and observer independence. This makes them fundamental cosmological laws and it is important to investigate the consequences of their coexistence at the same length-scales (see Baryshev et al.,1998 [2]; Gromov et al. 2001 [12]; Teerikorpi et al. 2005 [27]).

## 5 Conclusion

A cosmological model is in fact a particular solution of the gravity field equations. This is why the roots of the conceptual problems of modern cosmology considered above actually lie in the theory of gravitation. In fact, all fundamental forces in physics (strong, weak, electromagnetic) have quantum nature, (i.e. there are quanta of corresponding fields which carry the energy-momentum of the physical interactions), while GR is a non-quantum theory, which presents the geometrical interpretation of gravitational force (i.e. the curvature of space itself which is not material field in space) and exclude the concept of localizable gravitational energy. This is why the main problem of GR is the absence of the energy of the gravity field or pseudo-tensor character of gravity EMT (Landau& Lifshitz,1971 [19]). Together with GR the energy problem comes to cosmology and is the cause of the conceptual problems of SCM.

It is also possible that in cosmology we see just one example of a new physical phenomena where conservation laws are violated, receding velocities of whole galaxies may exceed the velocity of light and cosmological redshift is due to space expansion. Note that the explanation of the cooling of the photon gas in SCM, and hence the origin of the cosmic microwave background radiation, rest on the violation of the law of conservation of energy by the expanding space. However physics of "space creation" is still not tested in laboratory and hence needs more indirect observational evidence.

The big bang SCM is not the only possible model of the Universe. There are several cosmological models which are based on other fundamental hypotheses and give different interpretation of observable phenomena. A classification of possible relativistic cosmologies in accordance with basic initial assumptions were discussed by Baryshev et al.(1994) [1]. In particular relativistic quantum field approach to gravity, where the Minkowski space and conservation laws are valid, was considered by Feynman [9, 10]. Crucial observational tests of alternative cosmological models and gravity theories should be developed to understand real cosmological physics.

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